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Author's address: Maurus Rischatsch  
E-mail: [maurus.rischatsch@soi.uzh.ch](mailto:maurus.rischatsch@soi.uzh.ch)

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URL: [www.soi.uzh.ch](http://www.soi.uzh.ch)  
E-mail: [soilib@soi.uzh.ch](mailto:soilib@soi.uzh.ch)

# Simulating WTP Values from Random-Coefficient Models

Maurus Rischatsch\*  
Socioeconomic Institute, University of Zurich,  
Switzerland

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## Abstract

Discrete Choice Experiments (DCEs) designed to estimate willingness-to-pay (WTP) values are very popular in health economics. With increased computation power and advanced simulation techniques, random-coefficient models have gained an increasing importance in applied work as they allow for taste heterogeneity. This paper discusses the parametrical derivation of WTP values from estimated random-coefficient models and shows how these values can be simulated in cases where they do not have a known distribution.

**JEL-Classification:** C15, C25

**Keywords:** willingness-to-pay, discrete choice, simulation, random-coefficient models

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## Introduction

Discrete choice experiments (DCEs) have gained increasing importance in applied health economics. They allow to calculate willingness-to-pay (WTP) values for goods which are not (yet) available on the market or for public goods that have no market price. Recently, random-coefficient models have become popular in DCEs because they allow for taste heterogeneity among the respondents, e.g. Revelt and Train [1999], Hole [2008]. Increased computation power and advanced simulation techniques have fostered this development. Hensher and Greene [2003] summarize the state of practice of mixed logit models.

This paper provides an overview of the methods available for calculating WTP values from random-coefficient estimates. In the next section, we discuss the two most commonly used so-called

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mixing distributions. The subsequent section deals with the calculation of the WTP distribution resulting from random-coefficient models. Frequently, the mixing distributions employed do not permit to derive the WTP distribution parametrically. However, simulation provides a solution in these cases, as shown in two illustrative examples.

## Mixing Distributions

Estimation of random-coefficient model require the choice of a density defined over the possible coefficient values, the so-called mixing distribution. The two most commonly used are the normal ( $N$ ) and the lognormal ( $LN$ ) distributions (see Revelt and Train [1999], Hole [2008]). The univariate  $N$  has the form

$$f_N(\beta|\mu, \sigma) = \frac{1}{\sqrt{(2\pi)\sigma}} \exp\left(-\frac{(\beta - \mu)^2}{2\sigma^2}\right), \quad (1)$$

where  $\mu$  and  $\sigma$  denote the mean and the standard deviation, respectively. Since,  $\beta \sim N(\mu, \sigma^2)$  causes the corresponding attribute to have positive or negative marginal utility, this specification results in a conflict with maintained economic hypotheses. For instance, an attribute may constitute a normal good for some individuals and an inferior one for others. To avoid this conflict, many researchers prefer the  $LN$  alternative,

$$f_{LN}(\beta|\mu, \sigma) = \frac{1}{\beta\sqrt{(2\pi)\sigma}} \exp\left(-\frac{(\ln(\beta) - \mu)^2}{2\sigma^2}\right). \quad (2)$$

It is of advantage to keep in mind that the  $N$  and  $LN$  distributions are related in the following way. If  $b \sim N(\mu, \sigma^2)$ , then

$$\beta \sim \exp(b) \quad (3)$$

is  $LN$  distributed. For more details, see Poirier [1995].

## Derivation of WTP Values

As long as the attributes enter the utility function in linear form, WTP for attribute  $k$  is a constant given by

$$WTP_k = -\frac{\beta_k}{\beta_p}, \quad (4)$$

where  $\beta_k$  is the coefficient of the attribute of interest and  $\beta_p < 0$  belongs to the price attribute. With fixed-coefficient models, the  $\beta$ 's do not vary across individuals and the calculation of the WTP are straight forward applying Eq. (4).

In the case of random-coefficient models, Eq. (4) calls for dividing the two estimated mixing distributions by each other. In general, this division does not result in a well-specified distribution.

It has only a known form (*LN*) if both  $\beta_k$  and  $\beta_p$ , are *LN*. Evidently, this facilitates calculation of WTP values. Meijer and Rouwendal [2006] show that if  $\beta_i \sim \exp(b_i)$  and  $b_i \sim N(\mu_i, \sigma_i^2)$  for  $i = k, p$ , then

$$WTP_k = -\frac{\beta_k}{\beta_p} = -\frac{\exp(b_k)}{\exp(b_p)} = -\exp(b_k - b_p). \quad (5)$$

Hence,  $WTP_k$  is lognormally distributed because  $b_k - b_p$  is normally distributed.<sup>1</sup> Therefore, one obtains  $WTP_k \sim LN(\tilde{\mu}_k, \tilde{\sigma}_k^2)$  where  $\tilde{\mu}_k = \mu_k - \mu_p$  and  $\tilde{\sigma}_k^2 = \sigma_k^2 + \sigma_p^2$ .<sup>2</sup> Hence, one can use the means  $(\mu_k, \mu_p)$  and standard deviations  $(\sigma_k, \sigma_p)$  estimated from the random-coefficient model to derive the WTP distribution.

Unfortunately, the ratio of two mixing distributions of the  $N$  type does not result in parametric distribution. Therefore, the coefficient of the price attribute,  $\beta_p$ , is usually treated as nonstochastic to be able to divide the numerator  $N$  mixing distribution by a scalar (see Hole [2008]). Parametric calculation of the WTP values of random-coefficient models is not possible as long as not both coefficients –  $\beta_k$  and  $\beta_p$  – are *LN*-distributed. One solution to this problem is to simulate the WTP values using the estimated mixing distributions.<sup>3</sup> Most common software packages feature random numbers drawn from standard distributions such as  $N$  and *LN*. Thus, first one can draw random numbers from the  $N$  or *LN* mixing distribution (specified by the estimated parameters of the random-coefficient model). Using these draws the WTP values are then again given by dividing each single draw for  $\beta_k$  and  $\beta_p$  by each other according to Eq. (4).

## Some Illustrative Examples

We start with an example where both the coefficient of the  $k$ 'th attribute and the price attribute are *LN*. This permits a comparison between the parametrically derived and simulated WTP values. The estimated parameters of *LN*-mixing distributions  $\hat{\mu}_k$  and  $\hat{\sigma}_k$  are usually the mean and the standard deviation of the underlying normal distribution. Hence, we have to calculate the mean, median and standard deviation of the WTP values themselves. According to Train [2003] and Hole [2008], the statistics of the WTP values are given by

$$\begin{aligned} WTP_{\text{mean}} &= \exp\left(\tilde{\mu}_k + \frac{\tilde{\sigma}_k^2}{2}\right) \\ WTP_{\text{median}} &= \exp(\tilde{\mu}_k) \\ WTP_{\text{sd}} &= \exp\left(\tilde{\mu}_k + \frac{\tilde{\sigma}_k^2}{2}\right) \sqrt{\exp(\tilde{\sigma}_k^2) - 1}. \end{aligned} \quad (6)$$

where  $\tilde{\mu}_k$  and  $\tilde{\sigma}_k$  are the mean and the standard deviation of the *LN* distributed WTP values as seen above.

<sup>1</sup> The so-called convolution of two independent normally distributed random variables is a normal distribution. That means that the difference of two normally distributed random variables is normally distributed.

<sup>2</sup> The coefficients are assumed to be independent of each other and therefore, the covariance is zero and drops out in the calculation.

<sup>3</sup> Another way to obtain the WTP values in this case is to estimate individual-specific coefficients conditional on the individuals actual choice (see Train [2003] and Greene et al. [2005]).

Alternatively, one can simulate these statistics by sampling a sufficient high number of random draws from the mixing distributions of the coefficients  $\beta_k$  and  $\beta_p$ . In this example, we have

$$\beta_k \sim LN(\mu_k, \sigma_k^2), \quad \beta_p \sim LN(\mu_p, \sigma_p^2), \quad (7)$$

where  $\mu_k, \mu_p, \sigma_k^2$  and  $\sigma_p^2$  are the estimated parameters from the random-coefficient model. According to Drukker and Gates [2006], the more uniform the coverage over the domain of integration, the better the numerical approximation. Therefore, we compare random draws from the mixing distribution with so-called Halton draws (see Train [2003], Ch.9). Halton draws divide the zero to one range into segments of equal size. We use these draws to calculate the quantiles from the mixing distributions and simulate the WTP values from these draws.

The WTP values are calculated according to Eq. (4) for each single draw as the draws are independent from each other. In the case of Halton draws, they first have to be randomized as they are ordered by construction. The descriptive statistics of these individual values approximate the parametric values as the number of draws increases. Here, we set  $\mu_k = -0.8$ ,  $\mu_p = -3.5$ ,  $\sigma_k = 1.1$  and  $\sigma_p = 0.8$ .<sup>4</sup> Table 1 compares the parametrically and the simulated WTP values. It shows that the simulated values approximate the parametric values with  $10^4$  random draws and practically converge with  $10^6$  draws. The Halton draws perform slightly better, especially in the case of the standard deviation. The second example illustrates the simulation of WTP values if  $\beta_k \sim N$  and

Statistic	Parametric	Random Draws		Halton Draws	
		$10^4$ Draws	$10^6$ Draws	$10^4$ Draws	$10^6$ Draws
$WTP_k^{\text{mean}}$	37.525	37.419 (-0.28%)	37.513 (-0.03%)	37.413 (-0.30%)	37.526 (0.00%)
$WTP_k^{\text{median}}$	14.880	14.856 (-0.16%)	14.880 (0.00%)	14.887 (0.05%)	14.881 (0.01%)
$WTP_k^{\text{sd}}$	86.875	84.071 (-3.23%)	86.538 (-0.39%)	84.092 (-3.20%)	86.895 (0.02%)

The difference between parametric and simulated values (in %) are given in parentheses.

Table 1: Parametric and simulated  $WTP_k$  values

$\beta_p \sim LN$ , making a parametric derivation impossible. One can again draw random numbers from the specified  $N$  and  $LN$  distributions to obtain the statistics of the WTP values for attribute  $k$ . This time,  $\mu_k = -1.4$  and  $\sigma_k = 1.9$ . The distribution of the WTP values is depicted in Figure 1, with mean -63.85 (-63.83) CHF, median -37.5 CHF, and standard deviation 133.81 (133.71) CHF. The values are obtained from  $10^6$  Halton draws and  $10^6$  random draws (in parentheses). It is visible to the naked eye that the functional form of the resulting  $WTP_k$  distribution is neither  $N$  nor  $LN$ .

<sup>4</sup> These values are taken from an ongoing discrete choice analysis conducted at the Socioeconomic Institute of the University of Zurich.

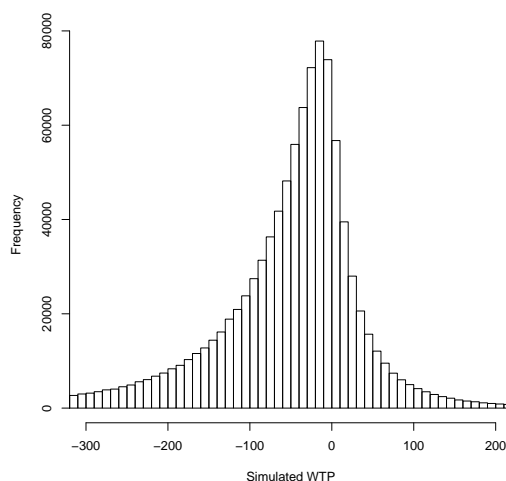


Figure 1: Histogram of simulated WTP values

## Conclusion

Random-coefficient models used in Discrete Choice Experiments (DCEs) permit the estimation of entire distributions of willingness-to-pay (WTP) values. However, WTP values amount to a ratio of two estimated mixing distributions, which presents problems unless both are lognormally distributed. Simulation using both random and Halton draws provides a solution to this. In both examples presented, especially the Halton draws perform very well. Thus, it is possible to obtain accurate WTP values for a normally distributed attribute while the price attribute is assumed to be lognormal by simulation.

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